



GCE A LEVEL MARKING SCHEME

SUMMER 2024

**A LEVEL
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

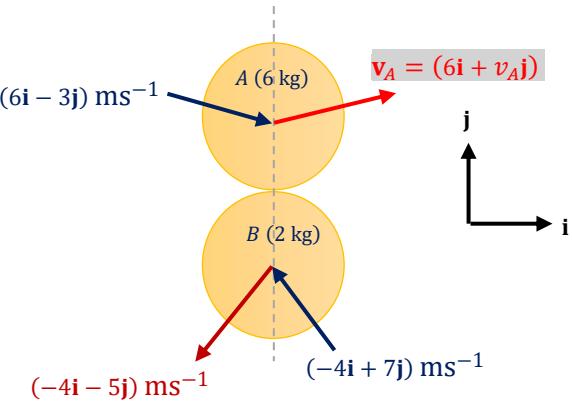
This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 6 FURTHER MECHANICS B

SUMMER 2024 MARK SCHEME

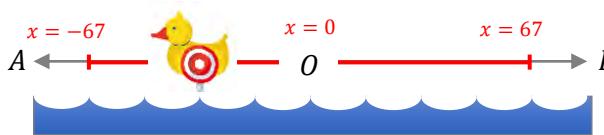
Q1	Solution	Mark	Notes
			Before collision After collision
(a)	<p>Con. of momentum (along line of centres)</p> $6(-3) + 2(7) = 6v_A + 2(-5)$ $(v_A = 1)$ $v_A = (6i + j) \text{ (ms}^{-1}\text{)}$	M1 A1 A1 [3]	Attempted All correct, oe $-4 = 6v_A - 10$ cao
	<p><u>Alternative Solution (Vector Method)</u></p> <p>Conservation of momentum</p> $6\mathbf{u}_A + 2\mathbf{u}_B = 6\mathbf{v}_A + 2\mathbf{v}_B$ $6(6\mathbf{i} - 3\mathbf{j}) + 2(-4\mathbf{i} + 7\mathbf{j}) = 6\mathbf{v}_A + 2(-4\mathbf{i} - 5\mathbf{j})$ $\mathbf{v}_A = (6\mathbf{i} + \mathbf{j}) \text{ (ms}^{-1}\text{)}$	(M1) (A1) (A1) ([3])	Attempted All correct, oe cao
(b)	<p>Restitution (along line of centres)</p> $(-5) - (1) = -e(7 - -3)$ <p>OR</p> $e = \frac{1 - -5}{7 - -3} = \frac{-5 - 1}{-3 - 7}$ <p>OR</p> $-e = \frac{-5 - 1}{7 - -3} = \frac{1 - -5}{-3 - 7}$ $e = \frac{3}{5} = 0.6$	M1 A1 A1 [3]	Attempted All correct, oe FT component j of \mathbf{v}_A from (a) cao

Q1	Solution	Mark	Notes
(c)	$\cos \theta = \frac{(-4\mathbf{i} + 7\mathbf{j}) \cdot (-4\mathbf{i} - 5\mathbf{j})}{ -4\mathbf{i} + 7\mathbf{j} -4\mathbf{i} - 5\mathbf{j} }$ $\cos \theta = \frac{16 - 35}{\sqrt{65}\sqrt{41}} \quad (= -0.368048451 \dots)$ $\theta = 112^\circ \quad (\text{nearest degree})$	M1 A1 A1 [3]	Use of (any form) $\cos \theta = \frac{\mathbf{u}_B \cdot \mathbf{v}_B}{ \mathbf{u}_B \mathbf{v}_B }$ oe cao, oe $(360^\circ - 112^\circ = 248^\circ)$
	<u>Alternative Solution</u> Sight of either one from BOTH rows $\theta = \tan^{-1}\left(\pm \frac{7}{4}\right) \text{ OR } \theta = \tan^{-1}\left(\pm \frac{4}{7}\right)$ $(\theta = \pm 60^\circ \dots) \quad (\theta = \pm 29^\circ \dots)$ $\theta = \tan^{-1}\left(\pm \frac{5}{4}\right) \text{ OR } \theta = \tan^{-1}\left(\pm \frac{4}{5}\right)$ $(\theta = \pm 51^\circ \dots) \quad (\theta = \pm 38^\circ \dots)$ $\theta = 60^\circ \cdot 255 \dots + 51^\circ \cdot 340 \dots$ $\theta = 112^\circ \quad (\text{nearest degree})$	(M1)	oe Correct calculation cao, oe $(360^\circ - 112^\circ = 248^\circ)$ ([3])
(d)	Impulse, $\mathbf{I} = \text{change in momentum}$ $(-20\mathbf{i} + 18\mathbf{j}) = 2\mathbf{v}_B - 2(-4\mathbf{i} - 5\mathbf{j})$ $\mathbf{v}_B = (-14\mathbf{i} + 4\mathbf{j}) \quad (\text{ms}^{-1})$	M1 A1 [2]	Difference in Momentum used, $(-20\mathbf{i} + 18\mathbf{j}) = -2\mathbf{v}_B + 2(-4\mathbf{i} - 5\mathbf{j})$ cao
Total for Question 1		11	

Q2	Solution	Mark	Notes
(a)	<p>Applying N2L (mg, $0 \cdot 2v^2$ opposing),</p> $1 \cdot 8g - 0 \cdot 2v^2 = 1 \cdot 8a$ <p>Dividing (by $0 \cdot 2, 1 \cdot 8$) and using $a = \frac{dv}{dt}$</p> $9 \frac{dv}{dt} = 9g - v^2$ $\frac{dv}{dt} = \frac{9g - v^2}{9}$	M1 A1 A1 [3]	Dimensionally correct equation Correct equation Convincing
(b)	$9 \int \frac{1}{9g - v^2} dv = \int dt$ $t (+C) = \begin{cases} \frac{9}{2 \times 3\sqrt{g}} \ln \left \frac{3\sqrt{g} + v}{3\sqrt{g} - v} \right \\ \frac{9}{3\sqrt{g}} \tanh^{-1} \left(\frac{v}{3\sqrt{g}} \right) \end{cases}$ <p>When $t = 0, v = \sqrt{g}$</p> $C = \begin{cases} \frac{9}{2 \times 3\sqrt{g}} \ln(2) \\ \frac{9}{3\sqrt{g}} \tanh^{-1} \left(\frac{1}{3} \right) \end{cases}$ <p>Using $v = 8$,</p> $t = \begin{cases} \frac{9}{2 \times 3\sqrt{g}} \ln \left \frac{3\sqrt{g} + 8}{3\sqrt{g} - 8} \right - \frac{9}{2 \times 3\sqrt{g}} \ln(2) \\ \frac{9}{3\sqrt{g}} \tanh^{-1} \left(\frac{8}{3\sqrt{g}} \right) - \frac{9}{3\sqrt{g}} \tanh^{-1} \left(\frac{1}{3} \right) \\ 0.878(03761 \dots) \end{cases}$ <p>Time taken = $0.878(03761 \dots)$ (s)</p>	M1 A1 A1 m1 A1 A1 [6]	Separating variables $\ln \left \frac{3\sqrt{g} + v}{3\sqrt{g} - v} \right $ or $\tanh^{-1} \left(\frac{v}{3\sqrt{g}} \right)$ Everything correct, oe Use of initial conditions Finding C Notes <ul style="list-style-type: none">These will be negative if $+C$ features on the RHS aboveThe numerator of 9 will not be present when the LHS is $\frac{t}{9}$ $t = \begin{cases} \frac{3}{2\sqrt{g}} \ln \left \frac{3\sqrt{g} + 8}{2(3\sqrt{g} - 8)} \right \\ \frac{3}{\sqrt{g}} \tanh^{-1} \left(\frac{8}{3\sqrt{g}} \right) - \frac{3}{\sqrt{g}} \tanh^{-1} \left(\frac{1}{3} \right) \end{cases}$ cao
(c)	<p>Using $a = v \frac{dv}{dx}$ to get</p> $v \frac{dv}{dx} = \frac{9g - v^2}{9}$ $9 \int \frac{v}{9g - v^2} dv = \int dx \quad \frac{9}{-2} \int \frac{-2v}{9g - v^2} dv = \int dx$ $-\frac{9}{2} \ln 9g - v^2 = x (+C)$ <p>When $x = 0, v = \sqrt{g}$</p> $-\frac{9}{2} \ln 8g = C$	M1 m1 A1 A1 m1	Used with DE Separating variables $\ln 9g - v^2 $ Everything correct Use of initial conditions

Q2	Solution	Mark	Notes
	$x = \begin{cases} -\frac{9}{2} \ln 9g - v^2 + \frac{9}{2} \ln 8g \\ \frac{9}{2} \ln \left \frac{8g}{9g - v^2} \right \end{cases}$ <p>When $v = 8$,</p> $x = \begin{cases} -\frac{9}{2} \ln 9g - 8^2 + \frac{9}{2} \ln 8g \\ \frac{9}{2} \ln \left \frac{8g}{9g - 8^2} \right \end{cases}$ $x = 5 \cdot 289(620824) \text{ (m)}$	A1	oe, for correct expression Notes <ul style="list-style-type: none"> • Signs will be reversed if $+C$ features on the LHS • The numerator of 9 will not be present when the RHS is $\frac{x}{9}$
Total for Question 2		A1 [7]	cao

Q3	Solution				Mark	Notes															
(a)	<table border="1"> <thead> <tr> <th>Shape</th><th>Area/mass</th><th>Distance from ACB</th><th>Distance from \perp</th></tr> </thead> <tbody> <tr> <td>Big</td><td>$\frac{1}{2}\pi(2a)^2\rho$ $(= 2\pi a^2\rho)$</td><td>$\frac{4(2a)}{3\pi}$ $(= \frac{8a}{3\pi})$</td><td>$\pm 2a$</td></tr> <tr> <td>Small</td><td>$\frac{1}{2}\pi a^2\rho$</td><td>$\pm \frac{4(a)}{3\pi}$ $(= \frac{4a}{3\pi})$</td><td>$\pm a$</td></tr> <tr> <td>Lamina</td><td>$\frac{5}{2}\pi a^2\rho$</td><td>\bar{x}</td><td>\bar{y}</td></tr> </tbody> </table>				Shape	Area/mass	Distance from ACB	Distance from \perp	Big	$\frac{1}{2}\pi(2a)^2\rho$ $(= 2\pi a^2\rho)$	$\frac{4(2a)}{3\pi}$ $(= \frac{8a}{3\pi})$	$\pm 2a$	Small	$\frac{1}{2}\pi a^2\rho$	$\pm \frac{4(a)}{3\pi}$ $(= \frac{4a}{3\pi})$	$\pm a$	Lamina	$\frac{5}{2}\pi a^2\rho$	\bar{x}	\bar{y}	Condone omission of ρ (mass per unit area)
Shape	Area/mass	Distance from ACB	Distance from \perp																		
Big	$\frac{1}{2}\pi(2a)^2\rho$ $(= 2\pi a^2\rho)$	$\frac{4(2a)}{3\pi}$ $(= \frac{8a}{3\pi})$	$\pm 2a$																		
Small	$\frac{1}{2}\pi a^2\rho$	$\pm \frac{4(a)}{3\pi}$ $(= \frac{4a}{3\pi})$	$\pm a$																		
Lamina	$\frac{5}{2}\pi a^2\rho$	\bar{x}	\bar{y}																		
					B1	B3 6															
					B1	B2 any 4 or 5, B1 any 2 or 3 correct															
					B1	Addition of areas/masses															
	<p>(i) Moments about AB</p> $\frac{5}{2}\pi a^2 \bar{x} = (2\pi a^2) \left(\frac{8a}{3\pi}\right) - \left(\frac{1}{2}\pi a^2\right) \left(\frac{4a}{3\pi}\right)$ $\bar{x} = \frac{28}{15\pi} a$				M1	Masses and moments consistent															
					A1																
					A1	Convincing, CSO															
	<p>(ii) Moments about \perp through A</p> $\frac{5}{2}\pi a^2 \bar{y} = (2\pi a^2)(2a) + \left(\frac{1}{2}\pi a^2\right)(a)$ <p>OR</p> $\frac{5}{2}\pi a^2 \bar{y} = (2\pi a^2)(-2a) + \left(\frac{1}{2}\pi a^2\right)(-a)$ $\bar{y} = \pm \frac{9}{5} a = \pm 1 \cdot 8a$ <p>Distance is $\frac{9}{5} a$ or $1 \cdot 8a$</p>				M1	Masses and moments consistent															
					A1																
					A1	cao, any form															
					[10]																
(b)	<p>Moments about A</p> $\frac{9}{5}a \times W = 4a \times T_2$ $T_2 = \frac{9}{20}W$ <p>(Required fraction is $\frac{9}{20}$)</p>					W = weight															
					M1																
					A1																
					A1	cao															
					[3]																
Total for Question 3					13																

Q4	Solution	Mark	Notes
(a)	Acceleration, $\frac{d^2x}{dt^2} = \pm \omega^2 x$ $1344 = \pm \omega^2 (\pm 84)$ $\omega^2 = 16$ $\omega = 4$ Period, $T = \frac{2\pi}{\omega}$ $= \frac{\pi}{2} \text{ (s)}$	M1 A1 A1 [3]	Used with $a = 1344$, $x = \pm 84$ cao Convincing
(b)	$v^2 = \omega^2(a^2 - x^2)$, $\omega = 4$, $x = \pm 84$, $v = \pm 52$ $(52)^2 = 4^2(a^2 - (84)^2)$ $a = 85$ Maximum speed = $a\omega = (85)(4)$ $= 340 \text{ (cms}^{-1}\text{)}$	M1 A1 A1 M1 A1 [5]	Used FT ω from (a) cao Used FT ω from (a) and corresponding a cao
(c)			
	Using $x = \pm a \cos(\omega t)$ or $x = \pm a \sin(\omega t + \frac{\pi}{2})$ $\pm 67 = \pm 85 \cos(4t)$ (Same/opposing signs) $4t = \cos^{-1}\left(\frac{67}{85}\right)$ or $4t = \cos^{-1}\left(\frac{-67}{85}\right)$ $4t = 0 \cdot 66286 \dots$ or $4t = 2 \cdot 47873 \dots$ $t = 0 \cdot 16571 \dots$ or $t = 0 \cdot 61968 \dots$ Times $= T + 0 \cdot 16571 \dots$, $T + 0 \cdot 61968 \dots$ $= 1 \cdot 73651 \dots$, $2 \cdot 190479 \dots \text{ (s)}$	M1 m1 m1 A1 A1 [5]	Allow $x = \pm a \sin(\omega t)$ FT $a > 67$ 1st correct method Remaining method Corresponding time for 1 st correct method. cao, both times
Total for Question 4		13	

Q5	Solution	Mark	Notes
			Length of rod = $2l$
(a)	<p>(i) Moments about A</p> $T \times 2l \sin 2\theta = W \times l \cos \theta$ $T \times 2l \times 2\sin \theta \cos \theta = W \times l \cos \theta$ $T = \frac{W}{4} \operatorname{cosec} \theta$ <p>(ii) Resolve vertically</p> $R + T \sin \theta = W$ $R + \frac{W}{4} \operatorname{cosec} \theta \sin \theta = W$ $R = \frac{3}{4} W$	M1 A1 m1 A1 M1 A1 m1 A1 	Dim. correct, no missing/extra terms All correct Use of $\sin 2\theta = 2\sin \theta \cos \theta$ Convincing Dim. correct equation with 3 terms Elimination of T cao [8]
(b)	<p>Resolve horizontally</p> $F = T \cos \theta$ $F = \frac{W}{4} \operatorname{cosec} \theta \cos \theta = \frac{W}{4} \cot \theta$ <p>Use of $F \leq \mu R$ with $\mu = \frac{\sqrt{3}}{3}$</p> $\frac{W}{4} \cot \theta \leq \frac{\sqrt{3}}{3} \times \frac{3}{4} W$ $\cot \theta \leq \sqrt{3} \quad \text{or} \quad \tan \theta \geq \frac{\sqrt{3}}{3}$ $\theta \geq 30^\circ$	M1 A1 M1 A1 A1 A1 	Dimensionally correct equation, no missing/extra terms si (or equality $F_{lim} = \mu R$) cao [5]
Total for Question 5		13	

Q6	Solution	Mark	Notes												
(a)	$(V\bar{x} =) \pi \int_0^b xy^2 dx$ $(V\bar{x} =) \pi \frac{a^2}{b^2} \int_0^b x(b^2 - x^2) dx$ $(V\bar{x} =) \pi \frac{a^2}{b^2} \int_0^b (b^2x - x^3) dx$ $(V\bar{x} =) \pi \frac{a^2}{b^2} \left[\frac{b^2}{2}x^2 - \frac{1}{4}x^4 \right]_0^b$ $(V\bar{x} =) \frac{1}{4}\pi a^2 b^2$ <p>Using $V = \frac{2}{3}\pi a^2 b$ and dividing to get</p> $\bar{x} = \frac{3}{8}b$	M1 A1 m1 A1 A1 [5]	Used All correct, oe At least one term correctly integrated AND sight of correct limits Convincing												
(b)	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Shape</th> <th style="text-align: center;">Mass</th> <th style="text-align: center;">Distance of COM from ground</th> </tr> </thead> <tbody> <tr> <td></td> <td>$\frac{2}{3}\pi \left(\frac{h}{4}\right)^2 h\rho \quad \left(= \frac{1}{24}h^3\pi\rho\right)$</td> <td>$50 + \frac{3h}{8}$</td> </tr> <tr> <td></td> <td>$\pi(25)^2 \times 50 \times 20\rho \quad (= 625000\pi\rho)$</td> <td>25</td> </tr> <tr> <td></td> <td>$\pi\rho \left[\frac{1}{24}h^3 + (25)^2 \times 50 \times 20 \right]$</td> <td>$\bar{h}, 50$</td> </tr> </tbody> </table> <p>Moments about horizontal ground</p> $\pi\rho \left(\frac{1}{24}h^3 \left(50 + \frac{3h}{8} \right) + (25)^2 \times 50 \times 20 \times 25 \right)$ $= \pi\rho \left(\frac{1}{24}h^3 + (25)^2 \times 50 \times 20 \right) \times \bar{h}$ <p>Using $\bar{h} = 50$ and rearranging to get</p> $\frac{h^4}{64} = 25^3 \times 50 \times 20$ $h = 177.82(7941 \dots)$	Shape	Mass	Distance of COM from ground		$\frac{2}{3}\pi \left(\frac{h}{4}\right)^2 h\rho \quad \left(= \frac{1}{24}h^3\pi\rho\right)$	$50 + \frac{3h}{8}$		$\pi(25)^2 \times 50 \times 20\rho \quad (= 625000\pi\rho)$	25		$\pi\rho \left[\frac{1}{24}h^3 + (25)^2 \times 50 \times 20 \right]$	$\bar{h}, 50$	B1 B1 B1 B1 M1 A1 m1 A1 [8]	Condone omission of ρ (mass per unit area) For Distance of COM For Mass Both correct Must be addition of volumes Notes $h^4 = 10^9$ $h = (10^9)^{\frac{1}{4}} = 10^{\frac{9}{4}}$ cao
Shape	Mass	Distance of COM from ground													
	$\frac{2}{3}\pi \left(\frac{h}{4}\right)^2 h\rho \quad \left(= \frac{1}{24}h^3\pi\rho\right)$	$50 + \frac{3h}{8}$													
	$\pi(25)^2 \times 50 \times 20\rho \quad (= 625000\pi\rho)$	25													
	$\pi\rho \left[\frac{1}{24}h^3 + (25)^2 \times 50 \times 20 \right]$	$\bar{h}, 50$													
(c)	One possible limitation identified.	E1 [1]	For example, <ul style="list-style-type: none"> • Modelling the tree as a (uniform) solid • Considering tree/pot to have constant density • Shape of tree perfectly matches the region R 												
Total for Question 6		14													